

Non-equilibrium collective dynamics in high-energy heavy ion collisions

Akihiko Monnai

RIKEN BNL Research Center

Nishina Center for Accelerator-Based Science, RIKEN

Lunch Seminar

10th October 2013, Brookhaven National Laboratory, NY

Overview

1. Introduction

- The quark-gluon plasma
- High-energy nucleus-nucleus collisions

2. Dissipative hydrodynamics of QGP

- Thermodynamic variables
- Relativistic dissipative hydrodynamics with charge densities
- Baryon stopping

3. Center domain structure in QGP

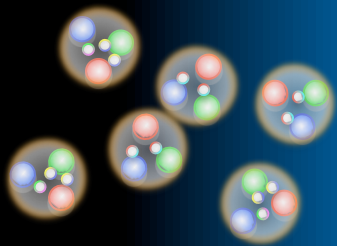
- Quark contribution to fluidity and opacity
- Discussion

4. Summary and outlook

Introduction

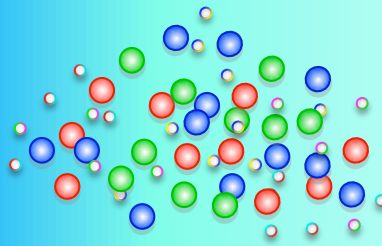
- **Quark-gluon plasma** (QGP): many-body system of deconfined quarks and gluons

Graphics by AM



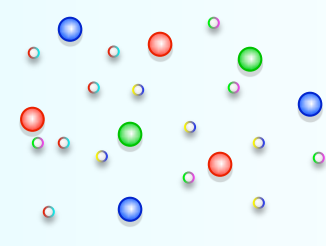
Hadron phase

(crossover)



sQGP

QGP phase

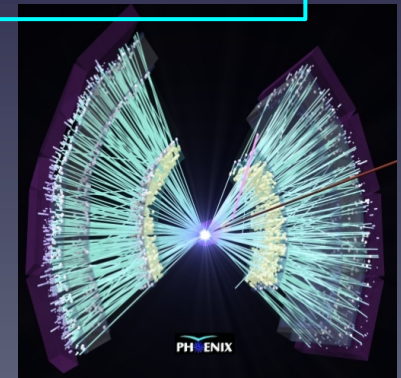


(wQGP?)

The QGP is supposed to have filled the early universe;
It can be produced in **heavy ion experiments** at RHIC & LHC

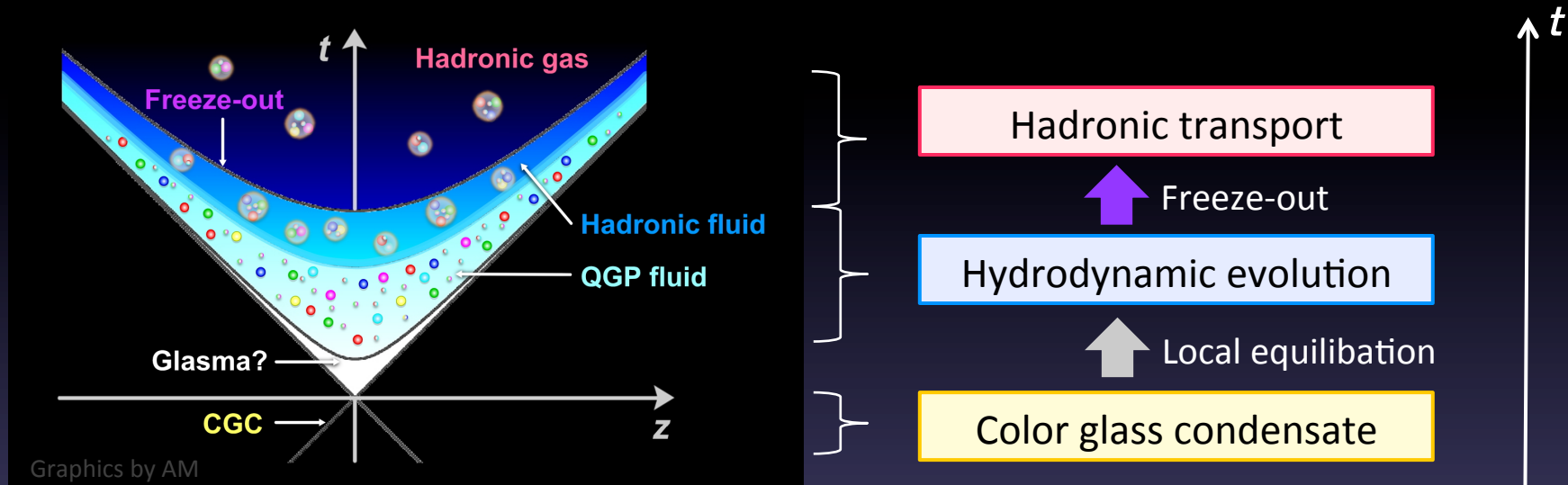
⇒ Heavy ion QGP is characterized with...

- **Near-perfect fluidity** (thermalized)
- **Large color opacity**



Introduction

■ “Standard model” of high-energy heavy ion collisions



► $\tau < 0$ fm/c: **color glass condensate** (saturated gluons)

► $\tau \sim 0$ -1 fm/c: **glasma?** (pre-equilibrated medium)

► $\tau \sim 1$ -10 fm/c: **QGP/hadronic fluid** (strongly-coupled medium)

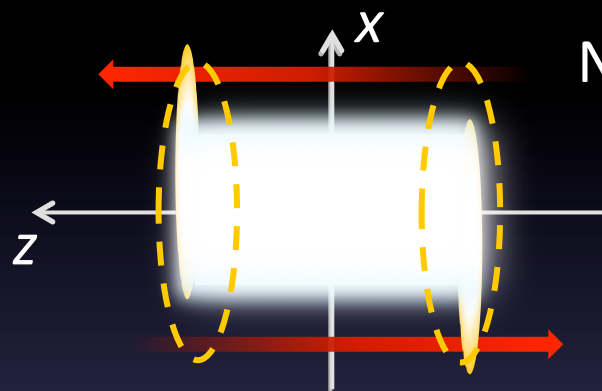
► $\tau > 10$ fm/c: **hadronic gas** (weakly-coupled medium)

2. Dissipative hydrodynamics of QGP

Reference: AM, Phys. Rev. C **86**, 014908 (2012)

Motivation

- Analyze hydrodynamic QGP at finite baryon density with **shear viscosity** + **bulk viscosity** + **baryon diffusion**



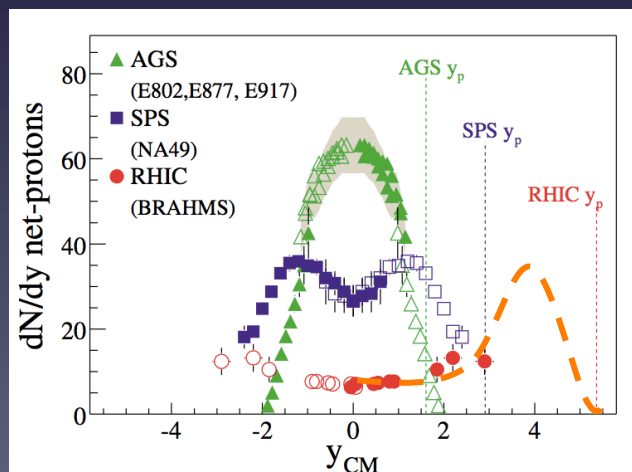
Net baryon is conserved at forward rapidity



- Precision physics including particle identification (p/\bar{p} ratio, etc.)
- Finite-density transport properties

- Baryon stopping

Plot: BRAHMS, PRL 93, 102301 (2004)

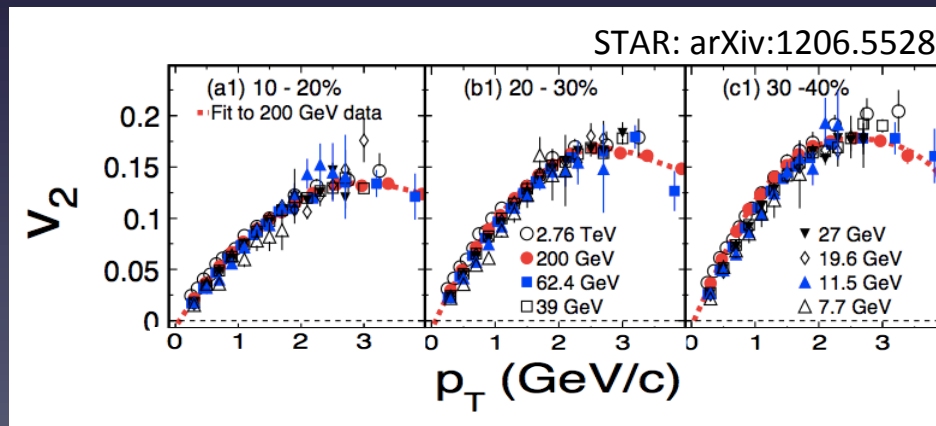
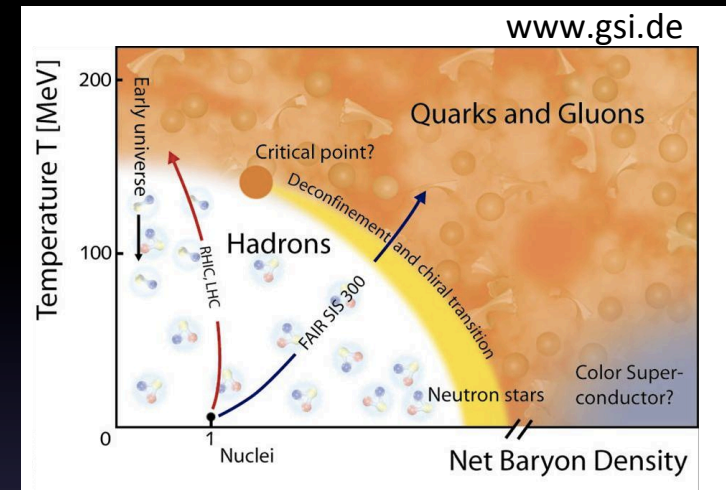


Baryon stopping can quantify kinetic energy available for QGP production

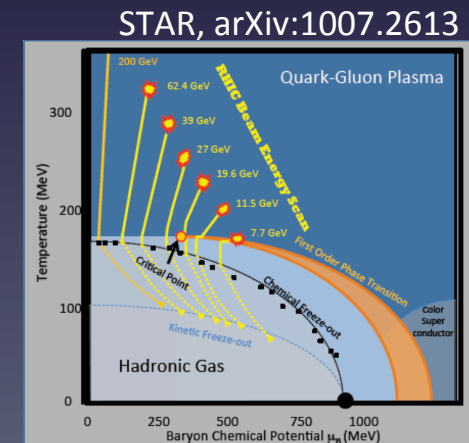
mean rapidity loss $\langle \delta y \rangle$
 = rapidity of projectile nuclei y_b
 – mean rapidity of net baryon $\langle y \rangle$

Motivation

- Exploring the **QCD phase diagram**
 - ▶ Finite baryon density is a *difficult* issue in first-principle calculations
 - ⇒ Hydrodynamics can be a help in the exploration
 - ▶ Beam energy scans for critical point search (at RHIC, FAIR, NICA, ...)



⇒ Large v_2 are observed at lower energies



Thermodynamic quantities

- In local rest frame $u^\mu = (1, 0, 0, 0)$

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$$

$$= \begin{pmatrix} e_0 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{pmatrix} + \begin{pmatrix} \delta e & W^x & W^y & W^z \\ W^x & \Pi + \pi^{xx} & \pi^{xy} & \pi^{xz} \\ W^y & \pi^{yx} & \Pi + \pi^{yy} & \pi^{yz} \\ W^z & \pi^{zx} & \pi^{yz} & \Pi + \pi^{zz} \end{pmatrix}$$

$$N_J^\mu = N_{J0}^\mu + \delta N_J^\mu \quad (J = 1, 2, \dots, N)$$

$$= \begin{pmatrix} n_{J0} \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \delta n_J \\ V_J^x \\ V_J^y \\ V_J^z \end{pmatrix}$$

2+N equilibrium quantities

Energy density: e_0

Hydrostatic pressure: P_0

J-th charge density: n_{J0}

10+4N dissipative currents

Energy density deviation: δe

Bulk pressure: Π

Energy current: W^μ

Shear stress tensor: $\pi^{\mu\nu}$

J-th charge density dev.: δn_J

J-th charge current: V_J^μ

Thermodynamic quantities

■ In general frame

$$T^{\mu\nu} = (e_0 + \delta e)u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}$$

$$N_J^\mu = (n_{J0} + \delta n_J)u^\mu + V_J^\mu$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

2+N equilibrium quantities

Energy density: e_0
 Hydrostatic pressure: P_0
 J-th charge density: n_{J0}

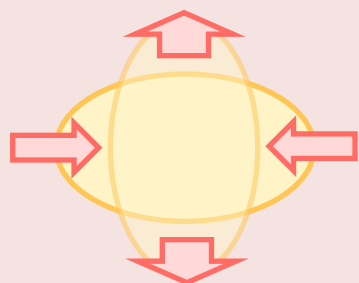
10+4N dissipative currents

Energy density deviation: δe
 Bulk pressure: Π
 Energy current: W^μ
 Shear stress tensor: $\pi^{\mu\nu}$
 J-th charge density dev.: δn_J
 J-th charge current: V_J^μ

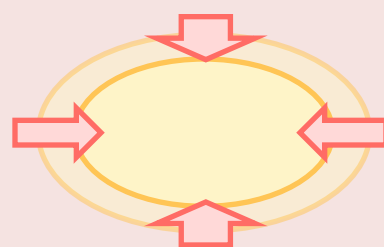
Thermodynamic quantities

■ Meaning of “dissipation” in fluids

Off-equilibrium processes at linear order

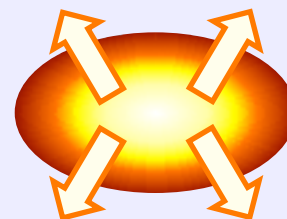


Shear viscosity
= response to
deformation

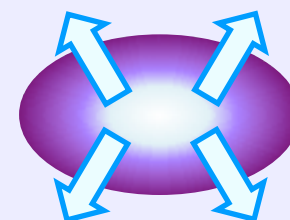


Bulk viscosity
= response to
expansion

viscosity



Energy dissipation
= response to
thermal gradient



Charge dissipation
= response to
chemical gradients

dissipation

- ▶ Cross terms among thermodynamic forces are present (discussed later)
- ▶ 2nd order corrections are required for hydrodynamic stability and causality

W. Israel, J. M. Stewart, Annals Phys 118, 341 (1979)

W.A. Hiscock, L. Lindblom, Phys. Rev. D 31, 725 (1985)

Dissipative hydrodynamics

■ Israel-Stewart theory with **net charges**?

► The entropy production


$$\partial_\mu s^\mu = - \sum_i \int \frac{g_i d^3 p_i}{(2\pi)^3 E_i} p_i^\mu \frac{\partial \phi}{\partial f^i} \partial_\mu f^i = \boxed{\sum_i \int \frac{g_i d^3 p_i}{(2\pi)^3 E_i} p_i^\mu y^i \partial_\mu f^i} \geq 0$$

where $\phi(f^i) \equiv f^i \ln f^i - \epsilon^{-1} (1 + \epsilon f^i) \ln(1 + \epsilon f^i)$, $f^i = \frac{1}{\exp(y^i) \mp 1}$

► Assume the deviation $\delta y^i = y^i - y_0^i$ is expressed as

$$\delta y^i = p_i^\mu q_i^J \varepsilon_\mu^J + p_i^\mu p_i^\nu \varepsilon_{\mu\nu} \quad (\text{Cf. } y_0^i = q_i^J \frac{\mu_J}{T} + p_i^\mu \frac{u_\mu}{T})$$

*Grad's moment method extended to **multi-conserved current** systems



$$\begin{aligned} \varepsilon_{\mu\nu} &= (B_\Pi \Pi + B_{\delta e} \delta e + \sum_J B_{\delta n_J} \delta n_J) \Delta_{\mu\nu} + (\tilde{B}_\Pi \Pi + \tilde{B}_{\delta e} \delta e + \sum_J \tilde{B}_{\delta n_J} \delta n_J) u_\mu u_\nu \\ &\quad + 2B_W u_{(\mu} W_{\nu)} + 2 \sum_J B_{V_J} u_{(\mu} V_{\nu)}^J + B_\pi \pi_{\mu\nu} \\ \varepsilon_\mu^J &= (D_\Pi^J \Pi + D_{\delta e}^J \delta e + \sum_K D_{\delta n_K}^J \delta n_K) u_\mu + D_W^J W_\mu + \sum_K D_{V_K}^J V_\mu^K \end{aligned}$$

Dissipative hydrodynamics

■ Israel-Stewart theory with **net charges**

► The entropy production up to the 2nd order

$$\begin{aligned}
 \partial_\mu s^\mu &= \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} (y_0 + \delta y) p_i^\mu \partial_\mu (f_0^i + \delta f_i) \\
 &= \sum_J \varepsilon_\nu^J \sum_i \int \frac{q_i^J g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\alpha \partial_\alpha f_0^i + \sum_J \varepsilon_\nu^J \sum_i \int \frac{q_i^J g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\alpha \partial_\alpha \delta f_i \\
 &\quad + \varepsilon_{\nu\rho} \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu p_i^\alpha \partial_\alpha f_0^i + \varepsilon_{\nu\rho} \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu p_i^\alpha \partial_\alpha \delta f_i
 \end{aligned}$$

Traditional I-S theory

equivalent to linear response theory

Semi-positive definite condition

$$\begin{aligned}
 \partial_\mu s^\mu &= \delta e D \frac{1}{T} - \Pi \frac{1}{T} \nabla_\mu u^\mu + W^\mu \left(\nabla_\mu \frac{1}{T} + \frac{1}{T} D u_\mu \right) \\
 &\quad + \pi^{\mu\nu} \frac{1}{T} \nabla_{\langle \mu} u_{\nu \rangle} - \sum_J \delta n_J D \frac{\mu_J}{T} - \sum_J V_J^\mu \nabla_\mu \frac{\mu_J}{T}
 \end{aligned}$$

2nd order dissipative
fluid dynamic equations

Dissipative hydrodynamics

■ Relativistic hydrodynamic equations

Conservation laws $\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu N_B^\mu = 0$

+

$$D = u^\mu \partial_\mu$$

$$\nabla^\mu = \partial^\mu - u^\mu D$$

The law of increasing entropy -> Constitutive equations

$$\begin{aligned} \Pi = & -\zeta \nabla_\mu u^\mu - \zeta_{\Pi \delta e} D \frac{1}{T} + \zeta_{\Pi \delta n_B} D \frac{\mu_B}{T} - \tau_\Pi D \Pi + \chi_{\Pi \Pi}^a \Pi D \frac{\mu_B}{T} + \chi_{\Pi \Pi}^b \Pi D \frac{1}{T} + \chi_{\Pi \Pi}^c \Pi \nabla_\mu u^\mu \\ & + \chi_{\Pi V}^a V_\mu \nabla^\mu \frac{\mu_K}{T} + \chi_{\Pi V}^b V_\mu \nabla^\mu \frac{1}{T} + \chi_{\Pi V}^c V_\mu D u^\mu + \chi_{\Pi V}^d \nabla^\mu V_\mu + \chi_{\Pi \pi} \pi_{\mu\nu} \nabla^{\langle \mu} u^{\nu \rangle} \end{aligned}$$

$$\begin{aligned} V^\mu = & \kappa_V \nabla^\mu \frac{\mu_B}{T} - \kappa_{VW} \left(\frac{1}{T} D u^\mu + \nabla^\mu \frac{1}{T} \right) - \tau_V \Delta^{\mu\nu} D V_\nu + \chi_{VV}^a V_K^\mu D \frac{\mu_B}{T} + \chi_{VV}^b V^\mu D \frac{1}{T} \\ & + \chi_{VV}^c V^\mu \nabla_\nu u^\nu + \chi_{VV}^d V_K^\nu \nabla_\nu u^\mu + \chi_{VV}^e V^\nu \nabla^\mu u_\nu + \chi_{V\pi}^a \pi^{\mu\nu} \nabla_\nu \frac{\mu_B}{T} + \chi_{V\pi}^b \pi^{\mu\nu} \nabla_\nu \frac{1}{T} \\ & + \chi_{V\pi}^c \pi^{\mu\nu} D u_\nu + \chi_{V\pi}^d \Delta^{\mu\nu} \nabla^\rho \pi_{\nu\rho} + \chi_{V\Pi}^a \Pi \nabla^\mu \frac{\mu_B}{T} + \chi_{V\Pi}^b \Pi \nabla^\mu \frac{1}{T} + \chi_{V\Pi}^c \Pi D u^\mu + \chi_{V\Pi}^d \nabla^\mu \Pi \end{aligned}$$

$$\begin{aligned} \pi^{\mu\nu} = & 2\eta \nabla^{\langle \mu} u^{\nu \rangle} - \tau_\pi D \pi^{\langle \mu\nu \rangle} + \chi_{\pi \Pi} \Pi \nabla^{\langle \mu} u^{\nu \rangle} + \chi_{\pi \pi}^a \pi^{\mu\nu} D \frac{\mu_B}{T} + \chi_{\pi \pi}^b \pi^{\mu\nu} D \frac{1}{T} + \chi_{\pi \pi}^c \pi^{\mu\nu} \nabla_\rho u^\rho \\ & + \chi_{\pi \pi}^d \pi^{\rho\langle \mu} \nabla_\rho u^{\nu \rangle} + \chi_{\pi V}^a V^{\langle \mu} \nabla^{\nu \rangle} \frac{\mu_B}{T} + \chi_{\pi V}^b V^{\langle \mu} \nabla^{\nu \rangle} \frac{1}{T} + \chi_{\pi V}^c V^{\langle \mu} D u^{\nu \rangle} + \chi_{\pi V}^d \nabla^{\langle \mu} V^{\nu \rangle} \end{aligned}$$

Simulation Setup

- Equation of state: **Lattice QCD** with Taylor expansion

$$\frac{P(T, \mu_B)}{T^4} = \frac{P(T, 0)}{T^4} + \frac{\chi_B^{(2)}(T, 0)}{2} \left(\frac{\mu_B}{T} \right)^2 + \mathcal{O} \left(\frac{\mu_B}{T} \right)^4$$

$P(T, 0)$: Equation of state at vanishing μ_B

S. Borsanyi *et al.*, JHEP 1011, 077

$\chi_B^{(2)}(T, 0)$: **2nd order baryon fluctuation**

S. Borsanyi *et al.*, JHEP 1201, 138

- Transport coefficients: **AdS/CFT** + **phenomenology**

Shear viscosity: $\eta = s/4\pi$

P. Kovtun *et al.*, PRL 94, 111601

Bulk viscosity: $\zeta = 5(\frac{1}{3} - c_s^2)\eta$

A. Hosoya *et al.*, AP 154, 229

Baryon dissipation: $\kappa_V = \frac{c_V}{2\pi} \left(\frac{\partial \mu_B}{\partial n_B} \right)_T^{-1}$

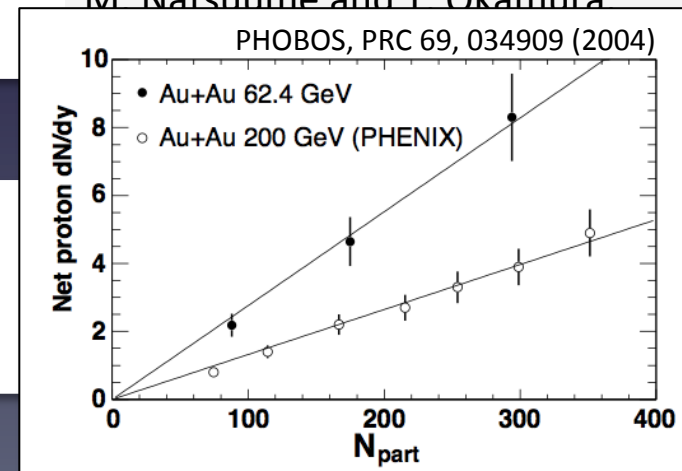
M. Natsuume and T. Okamura

- Initial conditions: **Color glass theory**

Energy density: MC-KLN

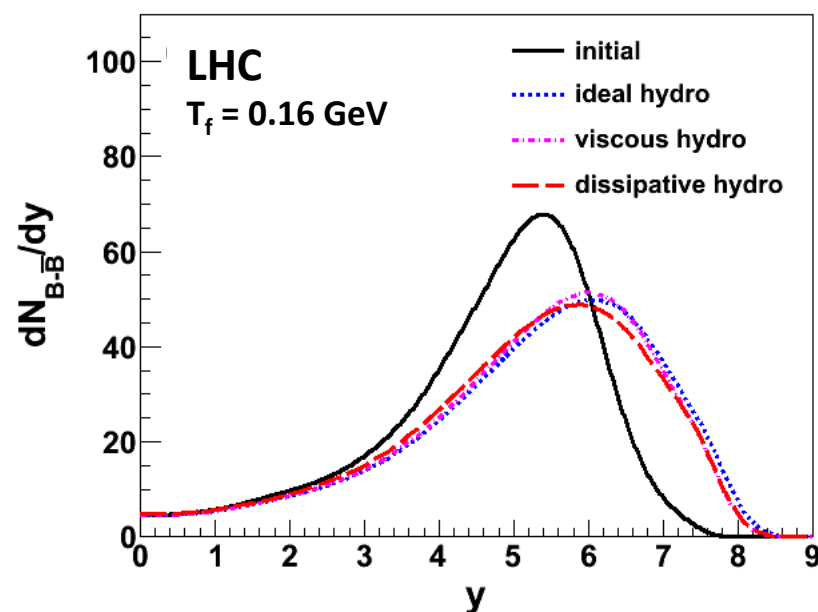
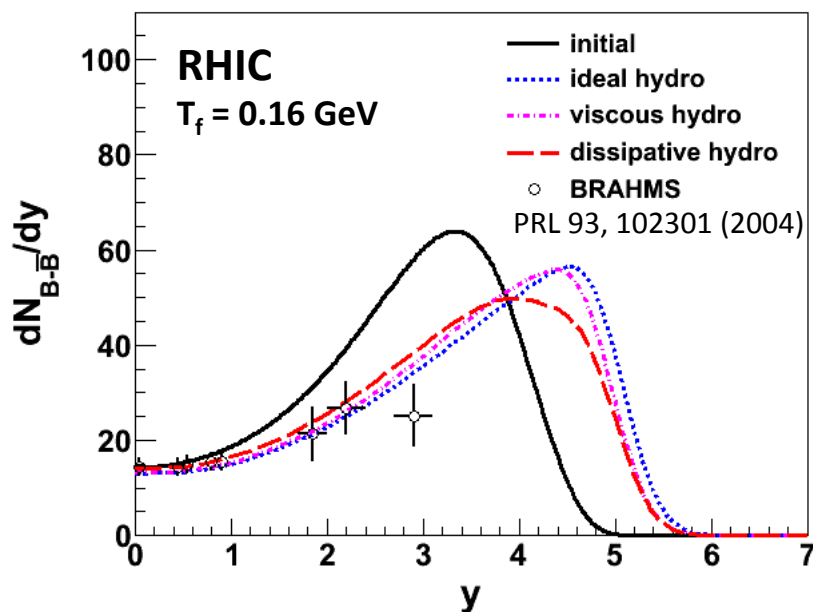
Net baryon density: Valence quark dist.

- Geometry: **(1+1)-D** expansion



Results

■ Net baryon rapidity distribution at RHIC and LHC



- Hydrodynamic evolution sends the net baryon number to forward rapidity
- Viscosities/dissipation could be non-negligible at RHIC

Results

■ Mean rapidity loss at RHIC

Mean rapidity loss $\langle \delta y \rangle = y_p - \langle y \rangle$

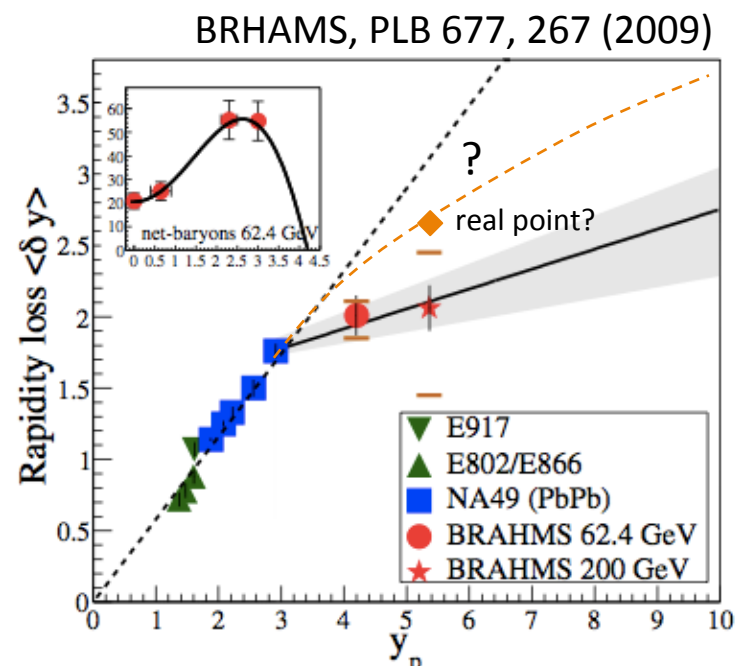
$$\langle y \rangle = \int_0^{y_p} y \frac{dN_{B-\bar{B}}(y)}{dy} dy \bigg/ \int_0^{y_p} \frac{dN_{B-\bar{B}}(y)}{dy} dy$$

Initial loss (RHIC): $\langle \delta y \rangle = 2.67$

Ideal hydro: $\langle \delta y \rangle = 2.09$

Viscous hydro: $\langle \delta y \rangle = 2.16$

Dissipative hydro: $\langle \delta y \rangle = 2.26$

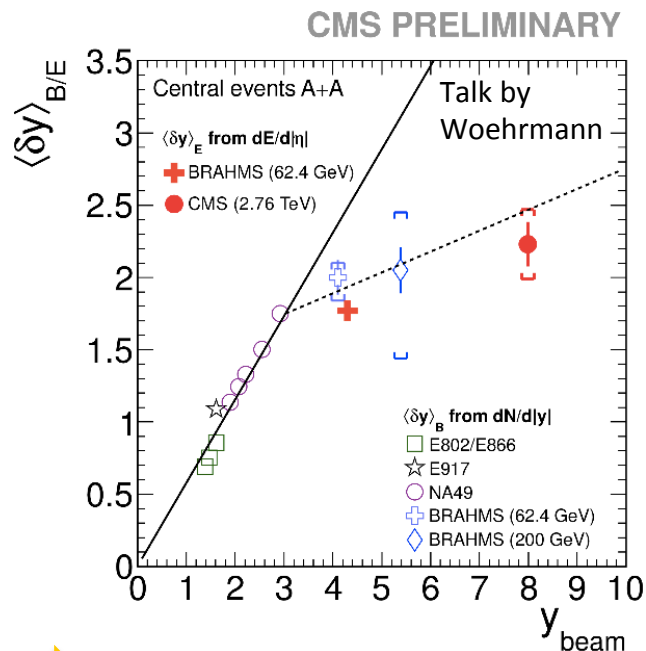


▪ Transparency of the collision is effectively enhanced in hydrodynamic evolution

➡ More kinetic energy is available for QGP production

Discussion

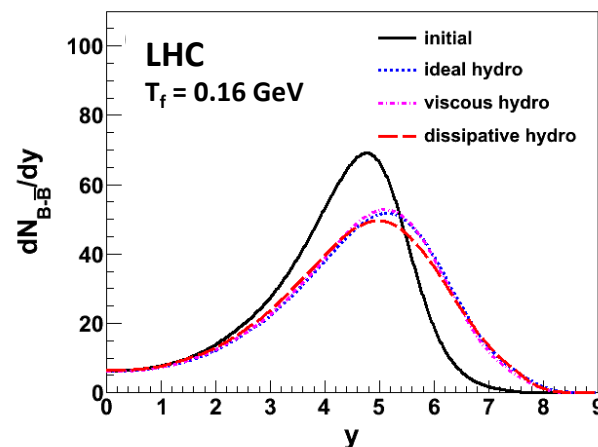
■ New implications from LHC



➡ More transparent initial conditions are preferred

Note: a *different* observable

$$\langle \delta y \rangle_E = \frac{2}{E_N N_{\text{part}}} \int_{-\infty}^{-y_{\text{beam}}} y' \frac{dE}{dy'} dy'$$



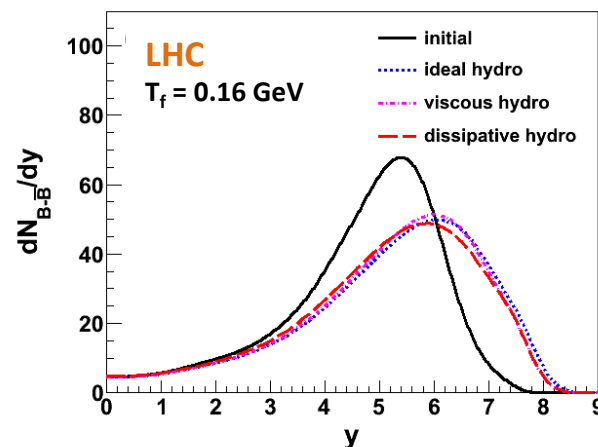
Initial loss: 3.88



Ideal hydro: 3.44

Viscous: 3.48

Dissipative: 3.51



Initial loss: 3.36



Ideal hydro: 2.81

Viscous: 2.86

Dissipative: 2.92

Cross-coupling effects (1)

■ Linear response theory and cross terms

Bulk pressure (w/o charges)

$$\Pi = \underbrace{-\zeta_{\Pi\Pi} \frac{1}{T} \nabla_\mu u^\mu}_{\text{Response to expansion}} - \underbrace{\zeta_{\Pi\delta e} D \frac{1}{T}}_{\text{Response to cooling}} = - \underbrace{\left(\frac{\zeta_{\Pi\Pi}}{T} + \frac{\zeta_{\Pi\delta e}}{T} c_s^2 \right)}_{\text{bulk viscosity } \zeta} \nabla_\mu u^\mu$$

- ▶ Response to expansion itself can be as **large** as shear viscosity
- ▶ Cancelled by the cross term except for crossover where $c_s^2 \sim 0$
- ➡ A reason for general smallness of bulk viscosity

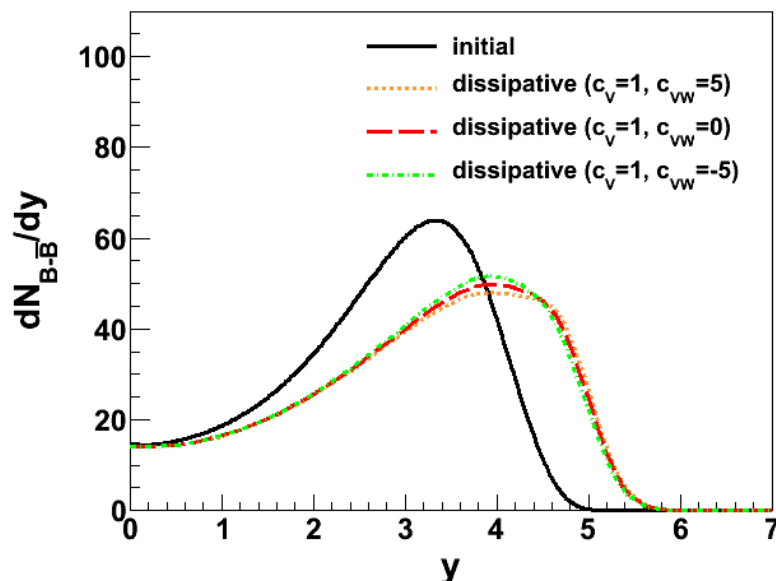
Baryon dissipation current

$$V^\mu = \kappa_V \nabla^\mu \frac{\mu_B}{T} - \kappa_{VW} \left(\nabla^\mu \frac{1}{T} + \frac{1}{T} D u^\mu \right)$$

- ▶ Baryon dissipation can be induced by **thermal gradient** + **acceleration**

Results

■ Thermo-diffusion effect (a.k.a. Soret effect)



- Baryon dissipation can be induced by **thermal gradients** (and acceleration)

$$V^\mu = \kappa_V \nabla^\mu \frac{\mu_B}{T} - \kappa_{VW} \left(\nabla^\mu \frac{1}{T} + \frac{1}{T} D u^\mu \right)$$

at the linear order

- The cross coefficient can be negative

$$\kappa_{VW} = c_{VW} \frac{n_{B0} T}{e_0 + P_0} \sqrt{5\eta T \kappa_V}$$

- The effect of cross coupling is likely to be small in high-energy collisions

because of the matter-antimatter symmetry

$$V^\mu(\mu_B) = -V^\mu(-\mu_B) \text{ which leads to } \kappa_{VW}(\mu_B = 0) = 0$$

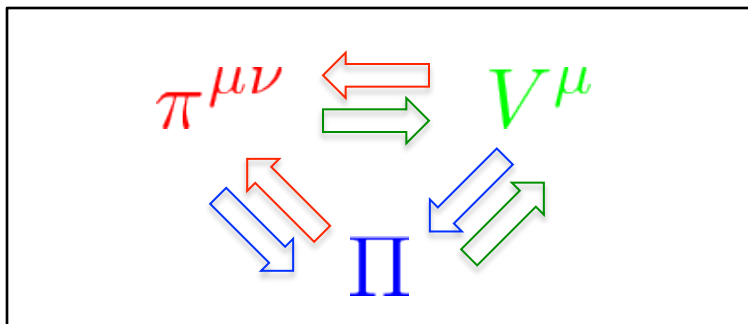
Cross-coupling effects (2)

■ Mixing of the currents at the 2nd order

System dependence

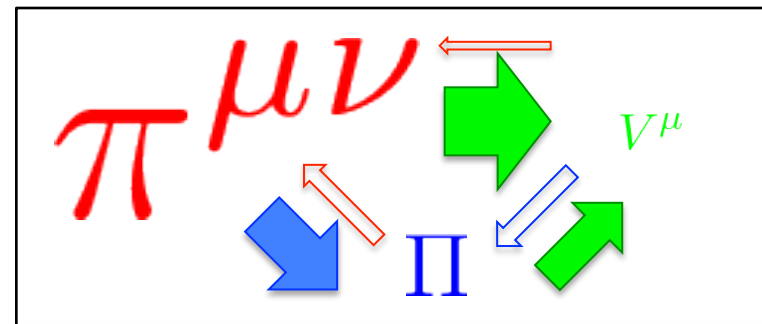
Hydrodynamic theory considers:

$$\pi^{\mu\nu} \sim \Pi \sim V^\mu$$



In high-energy nuclear collisions:

$$\pi^{\mu\nu} > \Pi > V^\mu$$



- ▶ Bulk-shear coupling term in bulk pressure
Baryon-shear and baryon-bulk coupling terms in baryon dissipation
have more impact than other 2nd order terms (numerically confirmed)
- ▶ Applicability of the expansion is dependent on the 2nd order transport coefficients

Brief summary

- Dissipative hydrodynamic model is developed and simulated at **finite baryon density**
 - ▶ Net baryon distribution is widened in hydrodynamic evolution
 - ⇒ Transparency of the collision is effectively enhanced
 - ⇒ **More kinetic energy** may be available at QGP production in early stage ($\sim 10\%$ at RHIC)
 - ▶ The results are sensitive to **baryon diffusion coefficient**
 - ⇒ Ambiguities remain in initial condition, but the distribution has important information
- Future prospects include:
 - ▶ Estimation of transverse expansion, inclusion of more realistic transport coefficients, etc.

2. Center domain structure in QGP

Reference: K. Kashiwa and AM, [arXiv:1309.6742 \[hep-ph\]](#)

Motivation

■ Center domain structure in the QGP

Asakawa, Bass and Müller, PRL 110,201301 (2013)

- ▶ One knows *how*, but not *why* hydrodynamics work so well
- ▶ Z_3 (center) symmetry in pure gauge system

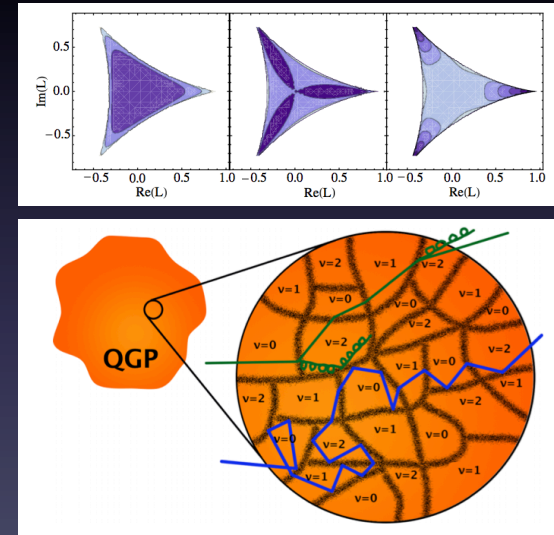
⇒ Three minima separated by energy barriers may exist in the complex plane of Polyakov-loop for the QGP phase

⇒ **Center domain structure** can develop in a QCD medium because CGC and glasma imply the typical size of correlation is $\sim 1/Q_s$

⇒ Mean free path is characterized by the **size of domain**; **small viscosity** and **large opacity** can be explained

- ▶ How does it approach pQCD picture at high T?

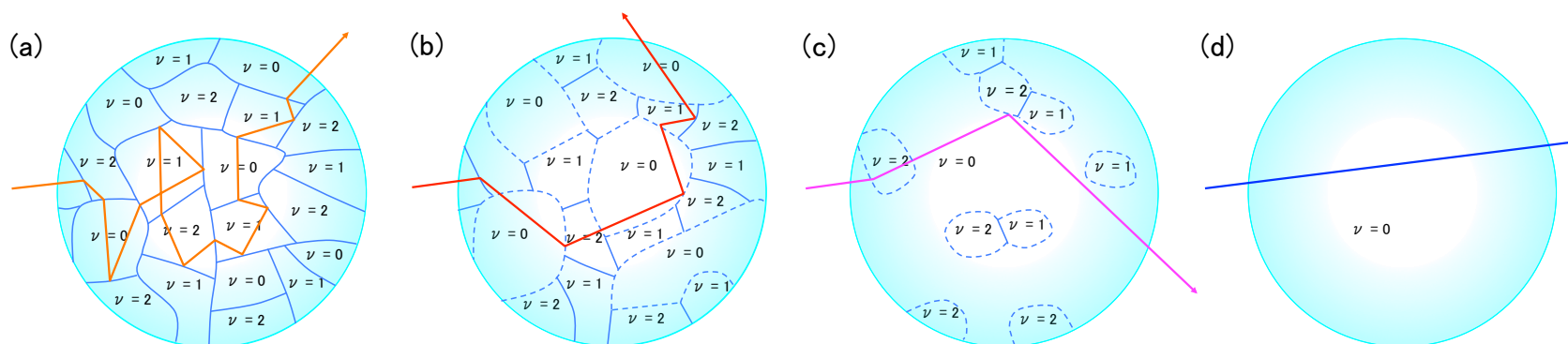
Asakawa, Bass and Müller, PRL 110,201301 (2013)



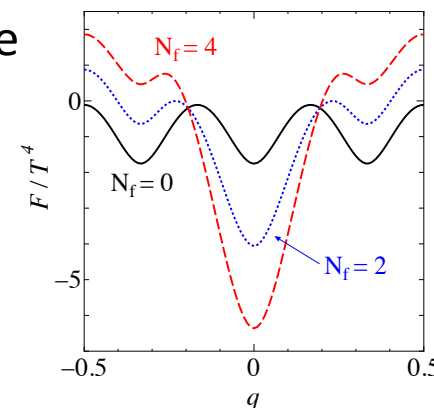
Center domain structure

■ Quark contribution

K. Kashiwa, AM, arXiv:1309.6742 [hep-ph]



- ▶ (a) → (b) Stable domains expand ($\nu=0$) while metastable ones ($\nu=1,2$) shrink due to pressure imbalance
 ⇒ Mean free path is **longer**, increasing viscosity
- ▶ (b) → (c) Percolation of stable domains
- ▶ (c) → (d) Metastable states vanish above a **critical temperature** $T_{\text{cri}} = T(P_{1,2} = 0)$; large viscosity and transparency
 ⇒ A smooth bridge from **sQGP (hydrodynamics)** to **wQGP (pQCD)**

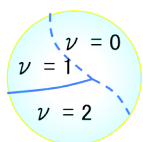


Center domain structure

■ Quark contribution

K. Kashiwa, AM, arXiv:1309.6742 [hep-ph]

► pp, pA and dA collisions

size of domains ~ 0.5 fm< radius of a proton ~ 0.8 fm

Several domains (and primordial fluidity) can be developed

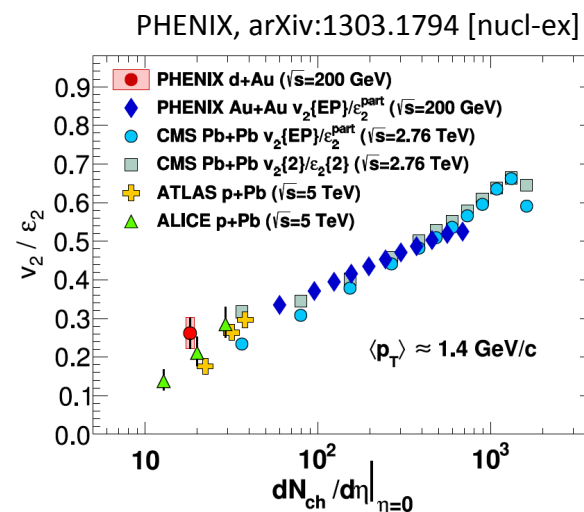
The structure might be fragile against finite size effects and quark contribution

► Heavy collisions at higher energies



Scenarios can be distinguished when above T_{cri} ($N_f \sim 3$)

Center domain is the origin of fluidity: yes \rightarrow small v_2 ($\nu = 0$ everywhere)
no \rightarrow large v_2 (in sQGP stage)



Summary and outlook

- Dissipative hydrodynamic model at **finite baryon density**
 - ▶ Baryon stopping is effectively reduced by hydrodynamic flow
 - ⇒ Energy available for QGP production could be larger
 - ▶ Net baryon distribution is sensitive to baryon diffusion
 - ▶ Future prospects: three dimensional analyses and more realistic transport coefficients for quantitative discussion, etc.
- Center domain structure with **quark contribution**
 - ▶ Provides a bridge from hydrodynamics to pQCD
 - ▶ A topological critical temperature may be present near $N_f \sim 3$
 - ▶ Future prospects: analyses on system size dependence, boundary effects, etc.

The end

- Thank you for listening!
- Website: <http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/>